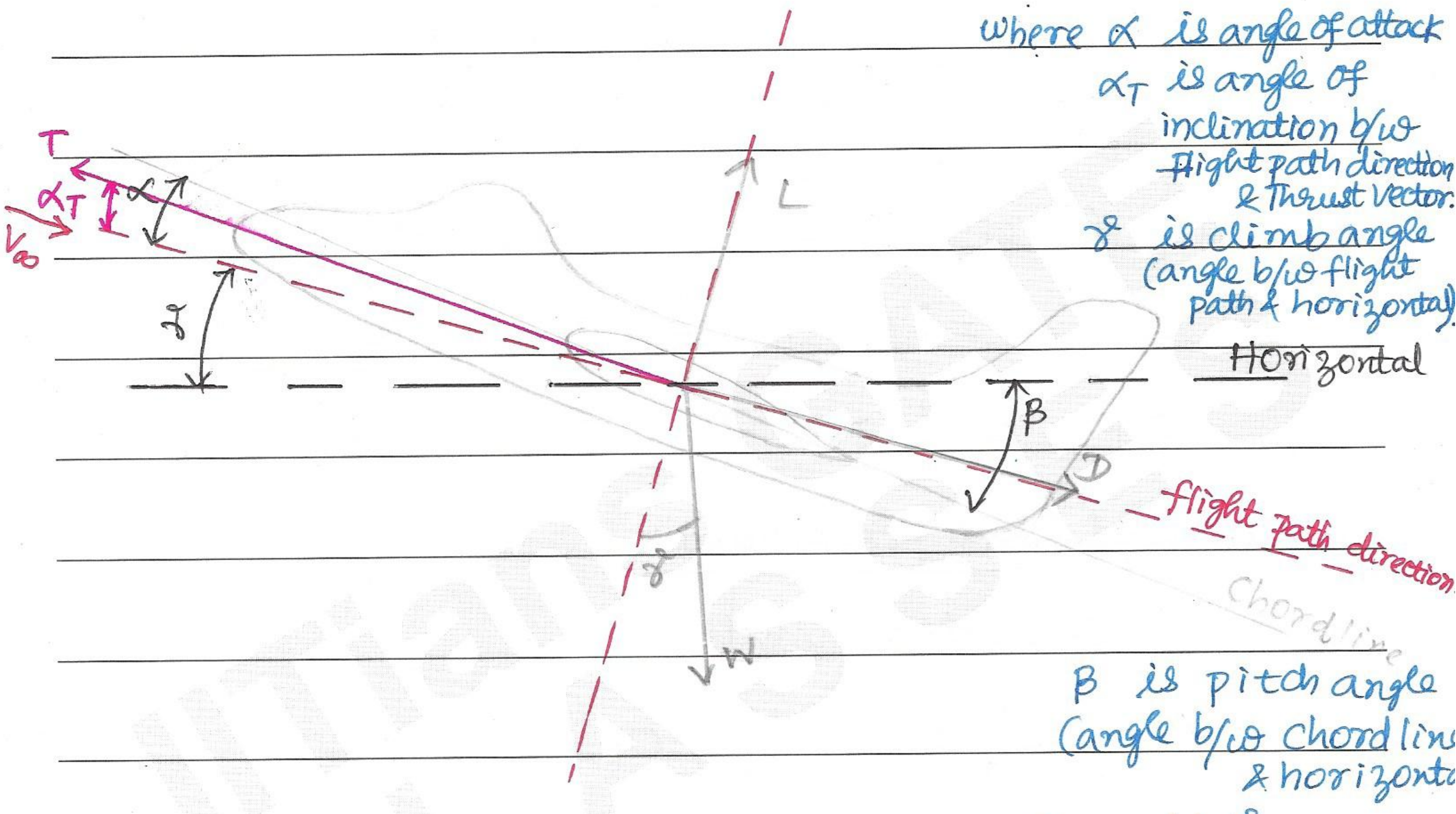
Airplane performance



where α is angle of attack
α_T is angle of inclination b/w flight path direction & Thrust vector.
γ is climb angle (angle b/w flight path & horizontal)

β is pitch angle (angle b/w chord line & horizontal)

$\therefore \beta = \alpha + \gamma$

i.e. Pitchangle = AOA + Climb angle

Four physical forces:

L = lift $\perp$r to flight path direction (relative wind)

D = Drag $\parallel^{el}$ to relative wind.

T = Thrust (at an angle α_T w.r.t. to flight path).

W = Weight $\perp$r to horizontal

J. Navaneetha Krishnan
M-Tech, IIST

According to Newton's second law (for curvilinear motion).

$$\Sigma F_{\parallel el} = m\frac{dv}{dt}, \quad \Sigma F_{\perp r} = \frac{mv^2}{r_c}$$

where r_c is radius of curvature.

Resolving forces $\parallel^{el}$ & $\perp r$ to flight path we get,

$$\Sigma F_{\parallel el} = T\cos\alpha_T - W\sin\gamma - D = m\frac{dv}{dt}$$

$$\Sigma F_{\perp r} = L - W\cos\gamma + T\sin\alpha_T = \frac{mv^2}{r_c}$$

Unaccelerated flight performance:

For unaccelerated flight,

$$\frac{dv}{dt} = 0, \quad \frac{v^2}{r_c} = 0.$$

Then,

$$T\cos\alpha_T - W\sin\gamma - D = 0$$

$$L - W\cos\gamma + T\sin\alpha_T = 0$$

In case of ~~steady~~ Straight & level flight,

$$\alpha_T = 0, \quad \gamma = 0$$

$$\therefore \quad T - D = 0 \; ; \; L - W = 0$$

$$\therefore \left.\begin{matrix}\boxed{T = D} \\ \boxed{L = W}\end{matrix}\right\} \text{conditions for straight \& level flight.}$$

1) condition of minimum drag:

$$L = W, \quad T = D$$

$$L/W = 1 = \frac{D}{T}$$

$$T = \frac{W}{(L/D)} \rightarrow (1)$$

$$(T_R)_{min} = D_{min} = \frac{1}{(L/D)_{max}} W$$

$C_D = a + bC_L^2$
where $a = C_{D,0}$
$b = \frac{1}{\pi e AR}$

For minimum drag, (L/D) should be maximum.

$$T_R = D = C_D \times \frac{1}{2}\rho_\infty V_\infty^2 S$$

$$= (a + bC_L^2) \times \frac{1}{2}\rho_\infty V_\infty^2 S$$

$$= \left(a \times \frac{1}{2}\rho_\infty S\right) V_\infty^2 + b\left[\frac{W^2}{\frac{1}{2}\rho_\infty S}\right]\frac{1}{V_\infty^2}$$

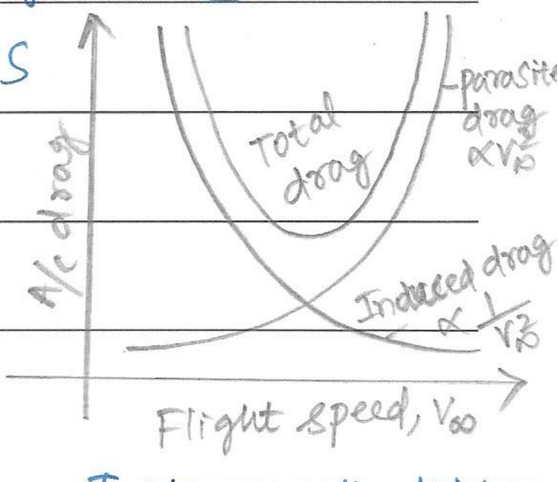
$L = W = C_L \times \frac{1}{2}\rho_\infty V_\infty^2 S$

$C_L = \frac{W}{\frac{1}{2}\rho_\infty V_\infty^2 S}$

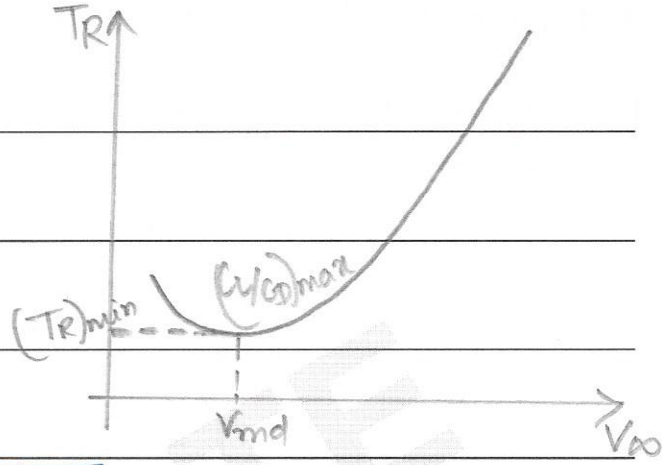
$$\boxed{T_R = \underbrace{K_1 V_\infty^2}_{\text{profile drag}} + \underbrace{\frac{K_2}{V_\infty^2}}_{\text{induced drag}}} \rightarrow (1A)$$

where $K_1 = a \times \frac{1}{2}\rho_\infty S$

$$K_2 = \frac{bW^2}{\frac{1}{2}\rho_\infty S}$$



J. Navaneetha Krishnan
M-Tech, IIST



$$\frac{dT_R}{dV_\infty} = 0$$

$$K_1(2V_\infty) - \frac{2K_2}{V_\infty^3} = 0.$$

$$V_\infty = \sqrt[4]{K_2/K_1}$$

$$\boxed{V_{md} = \sqrt[4]{b/a}\sqrt{\frac{2W}{\rho S}}} \rightarrow (1B)$$

Sub (1B) in (1A) we get

$$\boxed{D_{min} = 2W\sqrt{ab}} \rightarrow (1C)$$

$\Rightarrow$ minimum drag is independent of altitude

From eqn (1)

$$T_R = \frac{W}{(L/D)} = \frac{W}{(C_L/C_D)}$$

For T_R minimum, drag is minimum.

i.e. (C_L/C_D) is maximum, i.e. $\frac{C_D}{C_L}$ should be minimum i.e., $\dfrac{d \frac{C_D}{C_L}}{dC_L} = 0$ | $\dfrac{C_D}{C_L} = \dfrac{a+bC_L^2}{C_L}$

$$\frac{d(C_D/C_L)}{dC_L} = \frac{(a+bC_L^2).1 - C_L(2bC_L)}{C_L^2} = 0$$

$$\frac{a-bC_L^2}{C_L^2} = 0$$

$$a-bC_L^2 = 0$$

$$a = bC_L^2 \quad \text{i.e.} \quad \boxed{C_{D_0} = C_{D_i}}$$

$$\left.\begin{aligned} C_{L_{md}} &= \sqrt{a/b} \\ C_{D_{md}} &= 2a \\ (C_L/C_D)_{md} &= \frac{1}{2\sqrt{ab}} \end{aligned}\right\} \rightarrow (1D)$$

It is the condition for max endurance of jet engined a/c & Condition for max range of piston engined aircraft.

Navaneetha Krishnan J
M-Tech, IIST

② Minimum power required:

$$P_R = T_R \times V_\infty \quad \text{from eqn (1A)}$$

$$= \left[K_1 V_\infty^2 + \frac{K_2}{V_\infty^2}\right] V_\infty$$

$$\boxed{P_R = K_1 V_\infty^3 + \frac{K_2}{V_\infty}} \rightarrow \text{(2A)}$$

For P_R to be minimum,

$$\frac{dP_R}{dV_\infty} = 0$$

$$3K_1 V_\infty^2 - \frac{K_2}{V_\infty^2} = 0$$

$$3K_1 V_\infty^2 = \frac{K_2}{V_\infty^2}$$

$$V_\infty = \sqrt[4]{\frac{K_2}{3K_1}}$$

$$\boxed{V_{mp} = \sqrt[4]{\frac{b}{3a}} \sqrt{\frac{2W}{\rho_\infty S}}} \rightarrow \text{(2B)}$$

$$V_{mp} = \frac{1}{\sqrt[4]{3}} V_{md}$$

$$\boxed{V_{mp} = 0.7598\, V_{md}} \rightarrow \text{(2C)}$$

$$P_R = T_R \times V_\infty$$

$$= \frac{W}{(C_L/C_D)} \times \sqrt{\frac{2W}{\rho_\infty S C_L}}$$

$$P_R = \frac{W^{3/2}}{\left(\frac{C_L^{3/2}}{C_D}\right)} \sqrt{\frac{2}{\rho_\infty S}}$$

$$P_R \propto \frac{1}{\left(\frac{C_L^{3/2}}{C_D}\right)}$$

$$(P_R)_{min} \propto \frac{1}{\left(\frac{C_L^{3/2}}{C_D}\right)_{max}}$$

For minimum power required $\left(\frac{C_L^{3/2}}{C_D}\right)$ should be maximum i.e. $C_D/C_L^{3/2}$ should be minimum

For min P_R:

$$\frac{d(C_D/C_L^{3/2})}{dC_L} = 0.$$

$$\frac{C_D}{C_L^{3/2}} = \frac{a+bC_L^2}{C_L^{3/2}}$$

$$\frac{(a+bC_L^2)(\tfrac{3}{2}C_L^{1/2}) - C_L^{3/2}(2bC_L)}{C_L^3} = 0$$

Navaneetha Krishnan J
M-Tech, IIST

On simplification we get,

$3a = bC_L^2$ i.e. $\boxed{C_{D0} = \frac{1}{3} C_{Di}}$

$C_{Lmp} = \sqrt{\frac{3a}{b}}$; $C_{L\,mp} = \sqrt{3}\, C_{Lmd}$

$\boxed{C_{Lmp} = 1.732 C_{Lmd}}$

$C_{Dmp} = 4a$; $C_{Dmp} = 2(2a)$

$= 2(C_{Lmd})$

$\boxed{C_{Dmp} = 2 C_{Lmd}}$

$(L/D)_{mp} = \frac{\sqrt{3}}{2} \frac{1}{2\sqrt{ab}} = 0.866 (L/D)_{md}$

(2D)

It is the condition for max Endurance ~~of~~ for piston Engined aircraft.

③ Minimum drag to velocity ratio

w.k.t. $T_R = D = K_1 V_\infty^2 + \frac{K_2}{V_\infty^2}$

$$\boxed{\frac{D}{V_\infty} = K_1 V_\infty + \frac{K_2}{V_\infty^3}} \rightarrow (3A)$$

For min (D/V_∞),

$$\frac{d(D/V_\infty)}{dV_\infty} = 0$$

$$K_1 - \frac{3K_2}{V_\infty^4} = 0$$

$$V_\infty = \sqrt[4]{\frac{3K_2}{K_1}}$$

$$\boxed{V_{m(D/V_\infty)} = \sqrt[4]{\frac{3b}{a}}\sqrt{\frac{2W}{\rho_\infty S}}} \rightarrow 3B$$

$$V_{m(D/V_\infty)} = \sqrt[4]{3}\, V_{md}$$

$$\boxed{V_{m(D/V_\infty)} = 1.316\, V_{md}} \rightarrow 3C$$

$$\frac{D}{V_\infty} = \frac{W}{(C_L/C_D)} \frac{1}{V_\infty}$$

$$= \frac{1}{(C_L/C_D)} \frac{W}{\sqrt{\frac{2W}{\rho_\infty S C_L}}}$$

$$\frac{D}{V_\infty} = \frac{1}{\left(\frac{C_L^{1/2}}{C_D}\right)} \sqrt{\frac{W\rho_\infty S}{2}}$$

$$\left(D/V_\infty\right)_{min} \propto \frac{1}{\left(\frac{C_L^{1/2}}{C_D}\right)_{max}}$$

Navaneetha Krishnan J
M-Tech, IIST

For $(D/V)_{min}$ then $\left(\frac{C_L^{1/2}}{C_D}\right)$ should be maximum

i.e. $\left(C_D/C_L^{1/2}\right)$ is minimum.

$$\frac{d\left(C_D/C_L^{1/2}\right)}{dC_L} = 0$$

$$\frac{d\left(\frac{a+bC_L^2}{C_L^{1/2}}\right)}{dC_L} = \frac{(a+bC_L^2)\left(\tfrac{1}{2}C_L^{-1/2}\right) - C_L^{1/2}(2bC_L)}{C_L} = 0$$

On simplification we get,

$$3bC_L^2 = a \quad \text{i.e.} \quad \boxed{C_{D_0} = 3C_{D_i}}$$

$$C_{Lm(D/V_\infty)} = \sqrt{\frac{a}{3b}}$$

$$\boxed{C_{Lm(D/V_\infty)} = \frac{1}{\sqrt{3}}C_{Lmd} = 0.577C_{Lmd}}$$

$$C_{Dm(D/V_\infty)} = a + bC_L^2 = a + b\left(\frac{a}{3b}\right)$$

$$C_{Dm(D/V_\infty)} = \frac{4a}{3} = \frac{2}{3}(2a)$$

$$\boxed{C_{Dm(D/V_\infty)} = \frac{2}{3}C_{Dmd}}$$

$$\boxed{C_{Dm(D/V_\infty)} = 0.667C_{Dmd}}$$

3)