CF I3

Velocity of Sound in a fluid

A = Cross-Section area of pipe

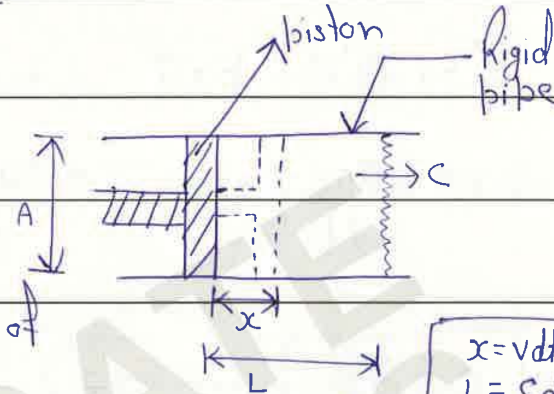
V = Velocity of piston

p = pressure of fluid in pipe before movement of piston

ρ = density of fluid before the moment of the piston

dt = small interval of time with which piston is moved

c = Velocity of pressure wave travelling in fluid



$$\boxed{\begin{array}{l} x = Vdt \\ L = cdt \end{array}}$$

Mass of fluid for a length 'L' before compression

$= \rho \times A \times L = \rho \times A \times c \times dt$

Mass of fluid after compression for length $(L - x)$

$= (\rho + d\rho) \times A \times (L - x)$

$= (\rho + d\rho) \times A \times (cdt - Vdt)$

from continuity

Mass of fluid before compression = Mass of fluid after compression

$$\rho A c dt = (\rho + d\rho) A (c - V) dt$$

$$\boxed{c\, d\rho = \rho V} \quad \text{—(1)} \qquad (\text{neglecting } V d\rho)$$

Net force on fluid element

$(p+dp)A - p \times A$ = mass per second $\times$ (change in velocity)

$$dp \times A = \frac{\rho A L}{dt}[V-0] = \frac{\rho A c\, dt}{dt} \times V$$

$$\boxed{c = \frac{dp}{\rho V}} \longrightarrow (2)$$

multiplying (1) & (2)

$$c^2 d\rho = dp$$

$$\boxed{c = \sqrt{\frac{dp}{d\rho}}}$$

<u>Sonic Velocity for an adiabatic process</u>

For adiabatic process $\frac{p}{\rho^\gamma} = C$

diff. above eq.

$$\frac{dp}{d\rho} = \frac{\gamma p}{\rho} \Rightarrow \text{Hence } \boxed{c = \sqrt{\gamma \frac{p}{\rho}} = \sqrt{\gamma R T}}$$

For isothermal Process : $\frac{p}{\rho}$ = constant

differentiate above eqn $\Rightarrow \frac{dp}{d\rho} = \frac{p}{\rho} = RT$

Hence

$$\boxed{C = \sqrt{RT}}$$

Imp. Points about Sonic velocity

(1) Sonic velocity is depend upon the change in density for a given change in pressure.

(2) It increase with growth in temperature

(3) Sonic velocity is higher in gases with a high value of gas constant. (R)

Mach Number (M) → defined as square root of the ratio of inertia force of a flowing fluid to the elastic force.

$$M = \sqrt{\frac{\rho A V^2}{KA}} \Rightarrow \boxed{M = \frac{V}{C}}$$

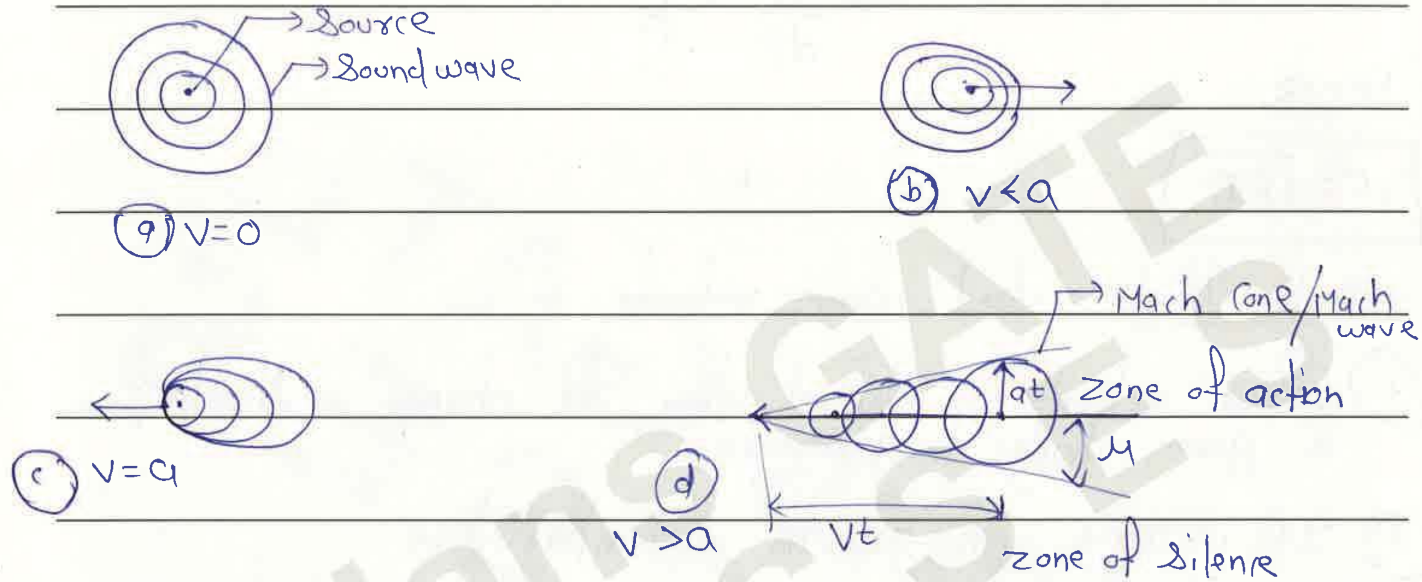
V → velocity of fluid
C → velocity of sound in the fluid.

(K = bulk Modulus)

$$\left\{ C = \sqrt{\frac{K}{\rho}} \right\}$$

M < 1 → Subsonic flow
M = 1 → Sonic flow
M > 1 → Supersonic flow

Mach Angle



Propagation of disturbance wave

(a), (b) → The disturbance wave reach a stationary observer before the source of disturbance could reach him in Subsonic flow

$$\sin\mu = \frac{at}{Vt} = \frac{a}{V} = \frac{1}{M}$$

$$\boxed{\mu = \sin^{-1}\left(\frac{1}{M}\right)}$$ Mach Angle

Compressible flow

Basic Equations:

① Equation of State:

$$\boxed{pV = mRT} \quad \text{—— (1)}$$

where, p = absolute pressure in N/m^2
V = volume occupied by mass (m) of the gas
ρ = mass density in kg/m^3
T = Absolute Temperature in Kelvin (K)
R = Gas Constant (287 J/kg-K)

② Continuity Equation: $\boxed{\rho A V = \text{Constant}}$ (for 1-D, steady flow)

differential form ⇒ $\boxed{\frac{dV}{V} + \frac{dA}{A} + \frac{d\rho}{\rho} = 0}$

③ Momentum Equation (Euler's Equation)

$$\boxed{\frac{dp}{\rho} + V dV + g dz = 0}$$

④ Energy Equation

Energy Equation → Incompressible flow
→ Compressible flow

Energy Equation (Bernoulli's Eq') for incompressible flow

$$\frac{dp}{\rho} + v\,dv + g\,dz = 0 \qquad \text{(Euler's Eq}^n\text{)}$$

integrating above eq.

$$\int \frac{dp}{\rho} + \int v\,dv + \int g\,dz = \text{constant}$$

$$\boxed{\frac{p}{\rho} + \frac{v^2}{2} + gz = \text{constant}}$$

For Compressible flow

(A) For isothermal process $\Rightarrow \frac{p}{\rho} = \text{constant} = C_1$

$$\rho = \frac{p}{C_1}$$

$$\int \frac{dp}{\rho} = \int \frac{dp}{p} C_1 = C_1 \int \frac{dp}{p} = C_1 \ln p = \frac{p}{\rho} \ln p$$

$$\int \frac{dp}{\rho} + \int v\,dv + \int g\,dz = \cancel{\text{constant}} \int 0$$

$$\Rightarrow \boxed{\frac{p}{\rho} \ln p + \frac{v^2}{2} + gz = \text{constant}}$$

Bernoulli's Equation for Adiabatic Process ($pV^{\gamma} = c$)

$$\frac{p}{\rho^{\gamma}} = c_1 \Rightarrow \rho = \left(\frac{p}{c_1}\right)^{1/\gamma}$$

$$\int \frac{dp}{\rho} = \int \frac{dp}{p^{1/\gamma}} c_1^{1/\gamma} = c_1^{1/\gamma}\left(\frac{p^{1-\frac{1}{\gamma}}}{1-\frac{1}{\gamma}}\right) = \left(\frac{\gamma}{\gamma-1}\right)\underbrace{\frac{c_1^{1/\gamma}}{p^{1/\gamma}}}_{\rho}\cdot p$$

$$= \left(\frac{\gamma}{\gamma-1}\right)\frac{p}{\rho}$$

Hence Substituting $\int \frac{dp}{\rho}$ into Euler's momentum equation

$$\int \frac{dp}{\rho} + \int V dV + \int g dz = \text{Constant}$$

$$\boxed{\left(\frac{\gamma}{\gamma-1}\right)\frac{p}{\rho} + \frac{V^2}{2} + gz = \text{constant}}$$

Further

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 = \left(\frac{\gamma}{\gamma-1}\right)\frac{p_2}{\rho_2} + \frac{V_2^2}{2} + gz_2$$

$$\left(\frac{\gamma}{\gamma-1}\right)\left\{\frac{p_1}{\rho_1} - \frac{p_2}{\rho_2}\right\} + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) = 0$$

use $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$, $\frac{p_1}{\rho_1} = RT_1$, $C_p = \frac{\gamma R}{\gamma - 1}$, $\frac{p_1}{\rho_1^\gamma} = \frac{p_2}{\rho_2^\gamma}$

above equation can be reduced to

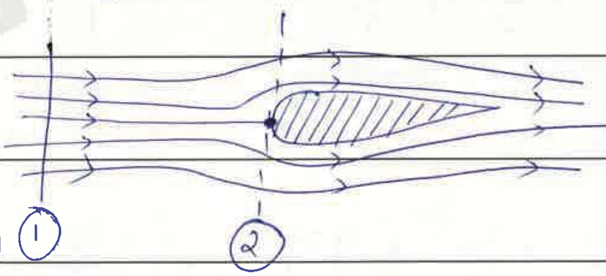
$$\boxed{c_p T_1 + \frac{V_2^2}{2} + gZ_2 = C_p T_1 + \frac{V_1^2}{2} + gZ_1 = \text{Constant}}$$

→ Steady flow energy equation
→ No Heat exchange
→ No shaft work

Stagnation Point / Stagnation Properties

① → p, T, V, ρ

② → P_0, T_0, ρ_0



$\boxed{V_0 = 0}$ (at stagnation Point)

using above equation with $Z_1 = Z_2$ at point ① & ②

$$c_p T + \frac{V^2}{2} = C_p T_0 + \cancel{\frac{V_0^2}{2}}^{\,0}$$

$$\boxed{h + \frac{V^2}{2} = h_0}$$

→ Total specific enthalpy

$$C_pT + \frac{V^2}{2} = C_pT_0$$

$$1 + \frac{1}{2}\frac{V^2}{C_pT} = \frac{T_0}{T} \qquad \left\{ C_p = \frac{\gamma R}{\gamma - 1} \right\}$$

$$\frac{T_0}{T} = 1 + \frac{1}{2}\frac{V^2}{T}\frac{\gamma-1}{\gamma R} \Rightarrow \frac{T_0}{T} = 1 + \frac{1}{2}(\gamma-1)\frac{V^2}{a^2} \qquad \left\{ a = \sqrt{\gamma R T} \right\}$$

$$\Rightarrow \boxed{\frac{T_0}{T} = 1 + \left(\frac{\gamma-1}{2}\right)M^2} \quad \text{IMP}$$

from adiabatic Relation $p_0 v_0^\gamma = p v^\gamma$ or $\frac{p_0}{\rho_0^\gamma} = \frac{p}{\rho^\gamma}$

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}} \Rightarrow \boxed{\frac{p_0}{p} = \left[1 + \left(\frac{\gamma-1}{2}\right)M^2\right]^{\frac{\gamma}{\gamma-1}}}$$

Similarly

$$\boxed{\frac{\rho_0}{\rho} = \left[1 + \left(\frac{\gamma-1}{2}\right)M^2\right]^{\frac{1}{\gamma-1}}}$$

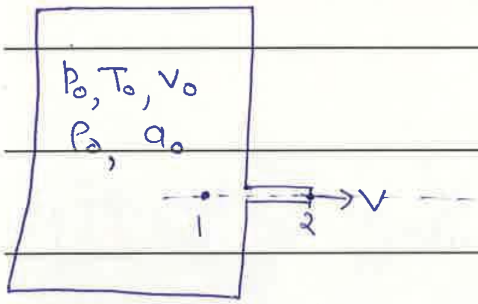
Relation between a and a_0

$$C_pT + \frac{V^2}{2} = C_pT_0$$

$$\frac{\gamma R T}{\gamma-1} + \frac{V^2}{2} = \frac{\gamma R T_0}{\gamma - 1}$$

$$\boxed{\frac{a^2}{\gamma-1} + \frac{V^2}{2} = \frac{a_0^2}{\gamma-1}} \quad \text{Imp}$$

Flow of Compressible fluid from a reservoir



At point ②

p_2, ρ_2, T_2, V_2

apply Bernoulli's eq^n at ① & ② (Assuming adiabatic process)

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_0}{\rho_0} + \cancel{\frac{V_0^2}{2}}^{\,0} = \left(\frac{\gamma}{\gamma-1}\right)\frac{p_2}{\rho_2} + \frac{V_2^2}{2}$$

$$V_2 = \sqrt{\left(\frac{2\gamma}{\gamma-1}\right)\frac{p_0}{\rho_0}\left(1 - \frac{p_2}{\rho_2}\frac{\rho_0}{p_0}\right)}$$

$$\frac{\rho_0}{\rho_2} = \left(\frac{p_0}{p_2}\right)^{\frac{1}{\gamma}}$$

Hence

$$\boxed{V_2 = \sqrt{\left(\frac{2\gamma}{\gamma-1}\right)\frac{p_0}{\rho_0}\left[1-\left(\frac{p_2}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}}$$

Imp

V_2 will be maximum when $p_2 = 0$ {for Given P_0, T_0, ρ_0}

$$V_2 = \sqrt{\frac{2\gamma}{\gamma-1}\frac{p_0}{\rho_0}} = a_0\sqrt{\frac{2}{\gamma-1}} = V_{max}$$

$$V_2 = \sqrt{2C_pT_0}$$