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<u>CHAPTER 1 – Network elements and sources</u>

Linearity, Linear Operator and Linearity in Electronics

Two properties needs to be satisfied for linearity:

a) Additivity : $f(x+y) = f(x) + f(y)$
b) Homogeneity : $f(ax) = af(x)$ where 'a' is scalar

The homogeneity and additivity properties together are called principle of superposition.

Steps to check for additivity :-

a) Give two inputs separately namely $x_1\ and\ x_2$. Response to the inputs are given as

$$y_1 = f(x_1)$$
$$y_2 = f(x_2)$$

b) Give single input $(x_1 + x_2)$. Response to the input is given as

$$y_{combined} = f(x_1 + x_2)$$

c) If $y_1 + y_2 = y_{combined}$, then add additivity is satisfied.
d) To summarize, if sum of response ($y_1 + y_2$) from individual inputs i.e. $x_1\ and\ x_2$ is equal to the combined response ($y_{combined}$) generated from sum of inputs ($x_1 + x_2$), then additivity is satisfied.

Q. Prove that if $f(x_1 + x_2) = f(x_1) + f(x_2)$ then $f(x_1 + x_2 + \cdots + x_n) = f(x_1) + f(x_2) + \cdots + f(x_n)$

Steps to check for homogeneity :-

a) Give input x_1. Response to the input is given as

$$y_1 = f(x_1)$$

b) Give input (ax_1). Response to the input is given as

$$y_{scaled} = f(ax_1)$$

c) If $y_{scaled} = ay_1$ i.e. Response to the scaled input is equal to the scaled output of the unscaled input.

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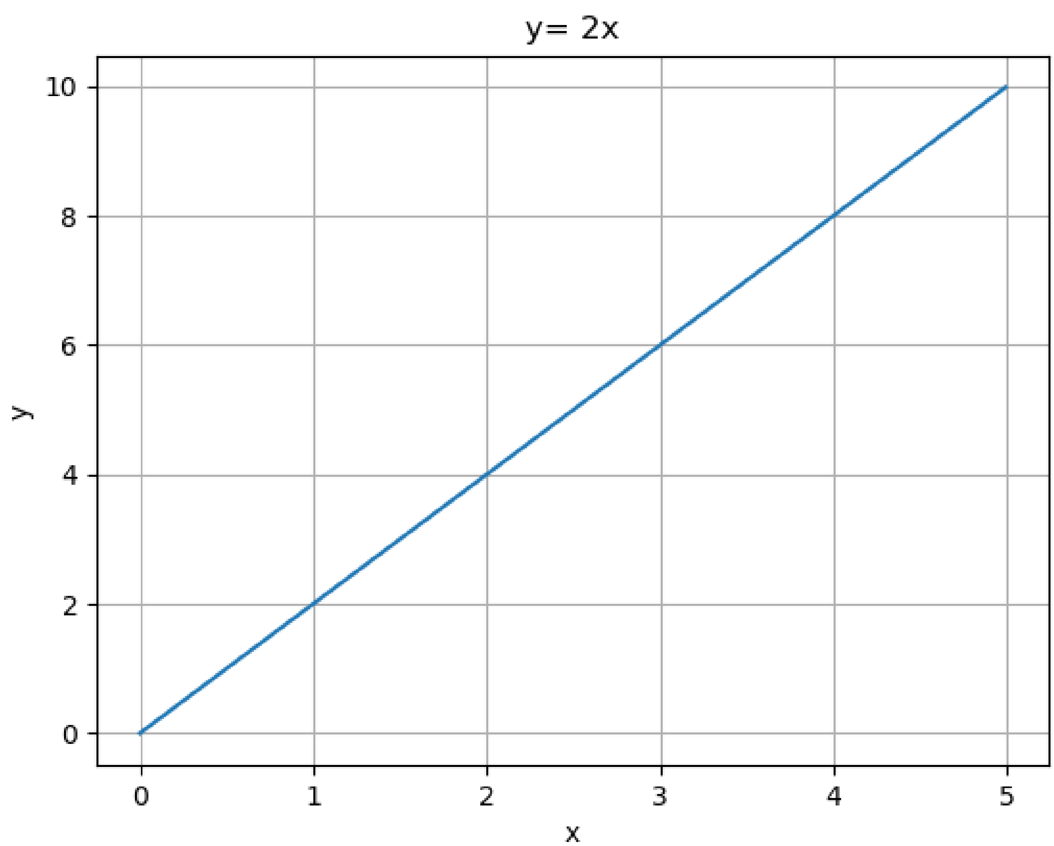
- <u>Linear function</u> :-
  1. $y = 10x$ is a linear function.
  2. $y = x^2$ is a non-linear function.
  3. $y = x + 1$ is a non-linear function.

  e.g. Graph of linear function



- <u>Linear operator</u> ($\underline{L}$) :-

  a) Additivity :-

$$\underline{L}\,(x + y) = \underline{L}(x) + \underline{L}(y)$$

  b) Homogeneity : -

$$\underline{L}\,(ax) = a\underline{L}(x)$$

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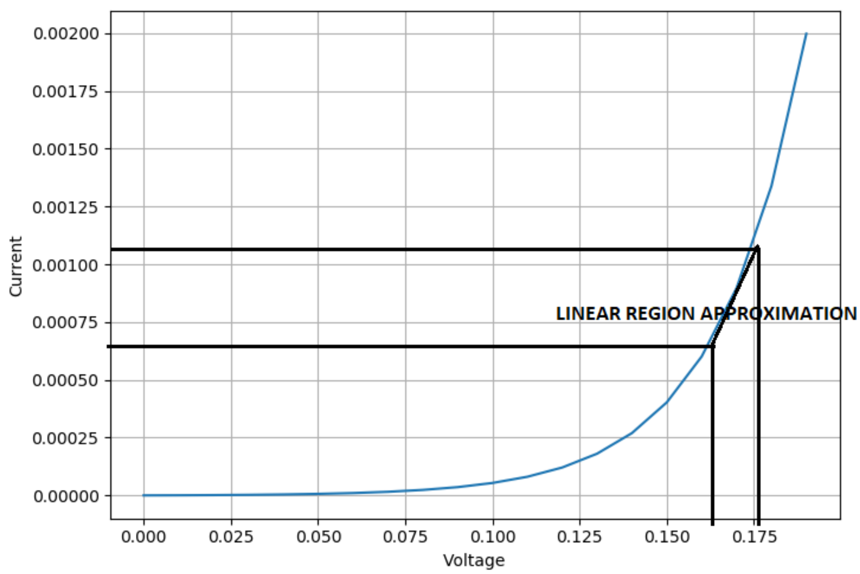
Examples

1. $\frac{d}{dx}$ is a linear operator
2. Double derivative is a linear operator
3. Curl, grad and divergence are linear operator
4. $x\frac{d}{dx}$ is non-linear operator

- Linearity in electronics :-
  - We make linear approximation around the Quiescent point or bias point.
  - We superimpose AC signal on DC biasing and approximate AC signal to be in the linear region.

  e.g.



Conclusion – This discussion is just meant to avoid certain confusion in the meaning of linearity. If someone says inductor is linear then it means that $\varphi = Li \; or \; \varphi \propto i$. Also, $V = L\,\frac{di}{dt}$ is linear i.e. if current $(i_1 + i_2)$ is flowing in an inductor then voltage across inductor will be $(v_1 + v_2)$. This does not mean that V is proportional to i in an inductor. In fact, V is proportional to rate of chance of current with respect to time in an inductor.

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Network Elements

Theory of networks start from a very fundamental question.

Q. All electric and magnetic phenomena can be explained or analysed in terms of fields, then why do we need another conceptual scheme?

Ans. Because we are engineers and we want to make our analysis simpler while maintaining certain amount of accuracy. With certain approximations/assumptions, we can avoid solving directly Maxwell's equations. Instead, we develop a conceptual scheme in terms of currents and voltages that are relatively simpler while maintaining high accuracy in low and mid range frequency.

These approximations are as follows, which allows us to work with circuit concepts and leave Maxwell's equations behind and are known as "Lumped matter approximation"–

1) The change of the magnetic flux outside the element = 0
2) The change of the charge in time inside conducting element = 0
3) Signal timescales of interest are much larger than propagation delay across the element (Low frequency)

Basic relationships (charge, voltage, current, flux, power, energy/work)

1) Current = rate change of charge w.r.t time

$$i = \frac{dq}{dt}$$

2) Voltage = Work per unit charge

$$V = \frac{W}{q}$$

If a differential amount of charge 'dq' is given a differential increase in energy dW then

$$V = \frac{dW}{dq}$$

3) Voltage = Time rate change of magnetic flux linkage

$$V = -\frac{d\varphi}{dt}$$

4) Power = Time rate change of work

$$P = \frac{dW}{dt} = \frac{dW}{dq} \times \frac{dq}{dt} = vi$$

5) Work = $\int P dt = \int vi\, dt$

1. Capacitors

A Capacitor is a passive electronic component which stores energy in the form of electrostatic field.

A) Basic relation for capacitor

$$q = cv$$

B) Current in a capacitor

$$i = \frac{dq}{dt} = \frac{dq}{dv} \times \frac{dv}{dt} = C\frac{dv}{dt} \;(where\ C = \frac{dq}{dv} = instantaneous\ capacitance)$$

$$if\ capacitance\ is\ function\ of\ time, i = c\frac{dv}{dt} + v\frac{dc}{dt}$$

For linear capacitors

$$i = C\frac{dv}{dt}\ where\ C\ is\ const.$$

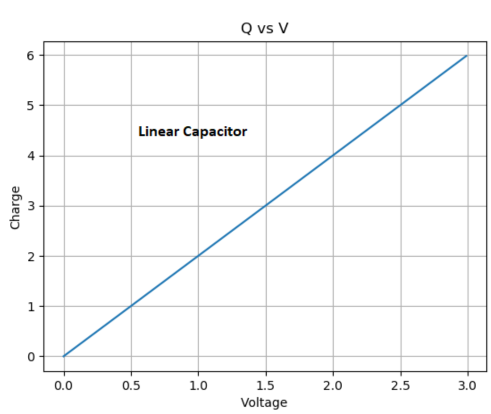
C) Instantaneous power = vi =v.C dv/dt

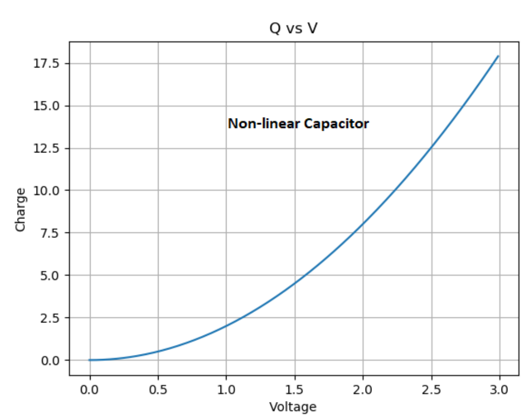
Work (Energy stored) = $\int v.C\frac{dv}{dt} = \frac{1}{2}Cv^2$

D) Instantaneous change in charge in a capacitor is not possible. For that infinite current would be required which is not possible in practical circuits.

i.e. $q_1 = q_2 \Rightarrow C_1 v_1 = C_2 v_2$

For a constant capacitive system, voltage across capacitor cannot change instantaneously





2. Inductors

An inductor is a passive electronic component that stores energy in the form of a magnetic field.

A) Basic relation for inductor

$$\varphi = Li \quad where\ \varphi = total\ magnetic\ flux = \iint \vec{B}.\overrightarrow{ds}$$

B) Voltage across inductor

$$v = \frac{d\varphi}{dt} = \frac{d\varphi}{di} \times \frac{di}{dt} = L\frac{di}{dt}\left(where\ L = \frac{d\varphi}{di}\right)$$

$$if\ inductance\ is\ function\ of\ time, v = L\frac{di}{dt} + i\frac{dL}{dt}$$

For linear Inductors

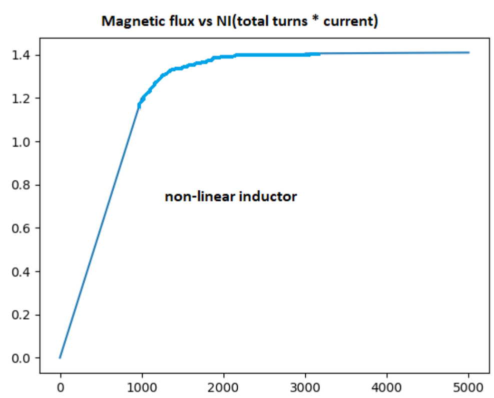
$$v = L\frac{di}{dt}$$

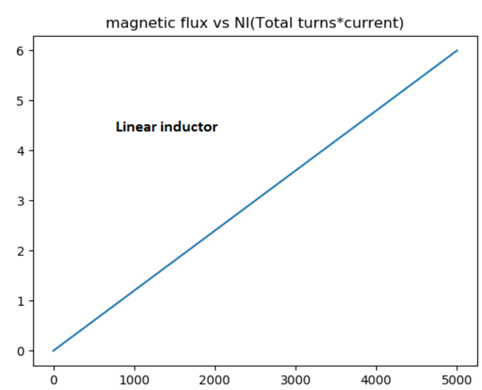
C) Instantaneous power

$$vi = iL\frac{di}{dt}$$

Work (Energy stored) = $\int i.L\frac{di}{dt} = \frac{1}{2}Li^2$

D) Instantaneous change in magnetic flux in an inductor is not possible. For that infinite voltage would be required which is not possible in practical circuits.
i.e. $\varphi_1 = \varphi_2 => L_1i_1 = L_2i_2$
For a constant inductive system, instantaneous change in current is not possible.





3. Resistances

A) Basic relation for the resistor

$$v = i\,R \; where \; R = \rho\frac{l}{A}\;(\rho = resistivity) = \frac{l}{\sigma A}\;(\sigma = conductivity)$$

$$i = Gv\;(G = Conductance)$$

B) Instantaneous power = vi

Work (Energy dissipated) = $\int vi\,dt = i^2R = v^2/R$

C) Dynamic resistance or ac resistance = $\frac{dv}{di}$

Dc resistance = $\frac{v}{i}$

e.g. Modeling of resistance in diode

Current in diode $I = I_0(e^{\frac{Vd}{\eta\, vt}} - 1)$

dc resistance = V/I

ac resistance = $\frac{dv}{di} = \frac{1}{\frac{di}{dv}} = \eta.vt/I_0.e^{\frac{Vd}{\eta\, vt}}$