GATE Computer Science Engineering Coaching by IGC
Disc Mathematics Assignment – 1

Q1. Indicate which of the following well - formed formula are valid :

a) $((P \Rightarrow Q) \wedge (Q \Rightarrow R)) \Rightarrow (P \Rightarrow R)$
b) $(P \Rightarrow Q) \Rightarrow (\neg P \Rightarrow \neg Q)$
c) $(P \wedge (\neg P \vee \neg Q)) \Rightarrow Q$
d) $((P \Rightarrow R) \vee (Q \Rightarrow R)) \Rightarrow ((P \vee Q) \Rightarrow R)$

Q2. Which of the following is a tautology:-

a) $(a \vee b) \rightarrow (b \wedge c)$
b) $(a \wedge b) \rightarrow (b \vee c)$
c) $(a \vee b) \rightarrow (b \rightarrow c)$
d) $(a \rightarrow b) \rightarrow (b \rightarrow c)$

Q3. The proposition $p \wedge (\sim p \vee q)$ is

a) A tautology
b) $\Leftrightarrow (p \wedge q)$
c) $\Leftrightarrow (p \vee q)$
d) A contradiction

Q4. What is the converse of the following assertion ?

"I stay only if you go."

a) I stay if you go.
b) If I stay then you go.
c) If you do not go then I do not stay.
d) If I do not stay then you go.

Q5. Consider two well-formed formulas in propositional logic :-

$F_1 : P \Rightarrow \neg P$

F_2 : $(P \Rightarrow \neg P) \vee (\neg P \Rightarrow P)$

Which of the following statements is correct ?

a) F_1 is satisfiable, F_2 is valid.
b) F_1 is unsatisfiable, F_2 is satisfiable
c) F_1 is unsatisfiable, F_2 is valid
d) F_1 and F_2 are both satisfiable

Q6. "If X then Y unless Z" is represented by which of the following formulas in propositional logic ?

("$\neg$" is negation, "$\wedge$" is conjunction, and " $\rightarrow$ " is implication)

a) $(X \wedge \neg Z) \rightarrow Y$
b) $(X \wedge Y) \rightarrow \neg Z$
c) $X \rightarrow (Y \wedge \neg Z)$
d) $(X \rightarrow Z) \wedge \neg Z$

Q7. Consider the following formula α and its two interpretations I_1 and I_2 .

$\alpha : (\forall x)\,[P_x \Leftrightarrow (\forall y)\,[Q_{xy} \Leftrightarrow \neg Q_{yy}]] \Rightarrow (\forall x)\,[\neg P_x]$

I_1 : Domain : the set of natural numbers
P_x = 'x is a prime number'
Q_{xy} = 'y divides x'
I_2 : Same as I_1 except that P_x = 'x is a composite number'

Which of the following statements are true ?

a) I_1 satisfies α, I_2 does not
b) I_2 satisfies α, I_1 does not
c) Neither I_1 nor I_2 satisfy α
d) Both I_1 and I_2 satisfy α

Q8. Consider the following logic program P

A(x) ← B(x,y), C(y)

← B(x,x)

Which of the following first order sentences is equivalent to P ?

a) $(\forall x)\ (\exists y)\ [[B(x,y) \wedge C(y)] \Rightarrow A(x)] \wedge \neg(\exists x)\ [B(x,x)]$

b) $(\forall x)\ (\forall y)\ [[B(x,y) \wedge C(y)] \Rightarrow A(x)] \wedge \neg(\exists x)\ [B(x,x)]$

c) $(\forall x)\ (\exists y)\ [[B(x,y) \wedge C(y)] \Rightarrow A(x)] \vee \neg(\exists x)\ [B(x,x)]$

d) $(\forall x)\ (\forall y)\ [[B(x,y) \wedge C(y)] \Rightarrow A(x)] \vee \neg(\exists x)\ [B(x,x)]$

Q9. The following propositional statement is $(P \rightarrow (Q \vee R)) \rightarrow ((P \wedge Q) \rightarrow R)$

a) Satisfiable but not valid
b) Valid
c) Contradiction
d) None of these

Q10. Which one of the first order predicate calculus statements given below correctly expresses the following english statement ?

"Tigers and lions attack if they are hungry or threatened."

a) $(\forall x)\ [(tiger(x) \wedge lion(x)) \rightarrow \{(hungry(x) \vee threatened(x)) \rightarrow attacks(x)\}]$

b) $(\forall x)\ [(tiger(x) \vee lion(x)) \rightarrow \{(hungry(x) \vee threatened(x)) \wedge attacks(x)\}]$

c) $(\forall x)\ [(tiger(x) \vee lion(x)) \rightarrow \{(hungry(x) \vee threatened(x)) \leftarrow attacks(x)\}]$

d) $(\forall x)\ [(tiger(x) \vee lion(x)) \rightarrow \{(hungry(x) \vee threatened(x)) \rightarrow attacks(x)\}]$

Q11. Consider the following propositional statements :-

P1 :- $((A \wedge B) \rightarrow C) = ((A \rightarrow C) \wedge (B \rightarrow C))$

P2 :- $((A \vee B) \rightarrow C) = ((A \rightarrow C) \vee (B \rightarrow C))$

Which one of the following is true ?

a) P1 is a tautology, but not P2
b) P2 is a tautology, but not P1
c) P1 and P2 are both tautologies
d) Both P1 and P2 are not tautologies.

Q12. Let Graph(x) be a predicate which denotes that x is a graph. Let Connected(x) be a predicate which denotes that x is connected. Which of the following first order logic sentences DOES NOT represent the statement: “Not every graph is connected.” ?

a) $\neg \forall x$ (Graph(x) $\Rightarrow$ Connected(x))

b) $\exists x$ (Graph(x) $\wedge \neg$ Connected(x))

c) $\neg \forall x(\neg$Graph(x) v Connected(x))

d) $\forall x$ (Graph(x) $\Rightarrow \neg$ Connected(x))

Q13. Which of the following is TRUE about formulae in Conjunctive Normal Form ?

a) For any formula, there is a truth assignment for which at least half the clauses evaluate to true.
b) For any formula, there is a truth assignment for which all the clauses evaluate to true.
c) There is a formula such that for each truth assignment at most one-fourth of the clauses evaluate to true.
d) None of these.

Q14. P and Q are two propositions, which of the following logical expressions are equivalent ?

1. P v $\neg$Q
2. $\neg(\neg$P $\wedge$ Q)
3. (P $\wedge$ Q) v (P $\wedge \neg$Q) v ($\neg$P $\wedge \neg$Q)
4. (P $\wedge$ Q) v (P $\wedge \neg$Q) v ($\neg$P $\wedge$ Q)

a) Only 1 and 2
b) Only 1, 2 and 3
c) Only 1, 2 and 4
d) All 1, 2, 3 and 4.

Q15. Which one of the following is the most appropriate logical formula to represent the statement :

“Gold and silver ornaments are precious.”

The following notations are used :-
G(x) : x is a gold ornament.
S(x) : x is a silver ornament.
P(x) : x is precious.

a) $\forall x\,(P(x) \rightarrow (G(x) \wedge S(x)))$

b) $\forall x\,((G(x) \wedge S(x)) \rightarrow P(x))$

c) $\exists x\,((G(x) \wedge S(x)) \rightarrow P(x))$

d) $\forall x\,((G(x) \vee S(x)) \rightarrow P(x))$

Q16. The binary operation $\square$ is defined as follows :-

P	Q	P $\square$ Q
T	T	T
T	F	T
F	T	F
F	F	T

Which one of the following is equivalent to P v Q ?

a) $\neg Q \square \neg P$

b) $P \square \neg Q$

c) $\neg P \square Q$

d) $\neg P \square \neg Q$

Q17. Consider the following well-formed formulae :-

1. $\neg \forall x(P(x))$
2. $\neg \exists x(P(x))$
3. $\neg \exists x(\neg P(x))$
4. $\exists x(\neg P(x))$

Which of the above are equivalent ?

a) 1 and 3
b) 1 and 4
c) 2 and 3
d) 2 and 4

Q18. Consider the following logical inferences.

I_1 : If it rains then the cricket match will not be played.
The match was played.
Inference :- There was no rain.

I_2 : If it rains then the cricket match will not be played.
It did not rain.
Inference :- The cricket match was played.

Which of the following is true ?

a) Both I_1 and I_2 are correct inferences.
b) I_1 is correct but I_2 is not a correct inference.
c) I_1 is not correct but I_2 is a correct inference.
d) Both I_1 and I_2 are not correct inferences.

Q19. What is the logical translation of the following statements ?

"None of my friends are perfect."

a) $\exists x\,(F(x) \wedge \neg P(x))$
b) $\exists x\,(\neg F(x) \wedge P(x))$
c) $\exists x\,(\neg F(x) \wedge \neg P(x))$
d) $\neg \exists x\,(F(x) \wedge P(x))$

Q20. Which of the following is NOT logically equivalent to $\neg \exists x(\forall y(\alpha) \wedge \forall z(\beta))$

a) $\forall x(\exists z(\neg \beta) \rightarrow \forall y(\alpha))$
b) $\forall x(\forall z(\beta) \rightarrow \exists y(\neg \alpha))$
c) $\forall x(\forall y(\alpha) \rightarrow \exists z(\neg \beta))$
d) $\forall x(\exists y(\neg \alpha) \rightarrow \exists z(\neg \beta))$

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Answers :-

1. A
2. B
3. B
4. A
5. A
6. A
7. D
8. C
9. A
10. D
11. D
12. D
13. D
14. B
15. D
16. B
17. B
18. B
19. D
20. A,D