CONSERVATION EQUATIONS

→ Continuity equation

→ Net mass flow rate through cross section 'S'

$$= \oiint_S \rho \vec{V}\, d\vec{S} \quad — (1)$$

→ Rate of decrement of mass within the control volume

$$= -\frac{\partial}{\partial t} \iiint \rho\, d\forall \quad — (2)$$

∴ By conservation of mass we have,

$$\boxed{\frac{\partial}{\partial t} \iiint \rho\, d\forall + \oiint_S \rho \vec{V} \cdot d\vec{S} = 0} \rightarrow \text{integral form}$$

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla(\rho V) = 0} \rightarrow \text{differential form}$$

for unsteady flow.

→ Momentum equation:

$$\boxed{\frac{\partial}{\partial t}\iiint \rho\vec{V}\,d\forall + \oiint_S (\rho\vec{V}\cdot d\vec{s})\vec{V} = \underbrace{-\oiint_S P\,d\vec{s}}_{\text{Pressure forces}} + \underbrace{\iiint \rho\vec{f}\cdot d\forall}_{\text{Body forces}} + F_{viscous}}$$

↳ Integral form.

→ $$\boxed{\frac{\partial}{\partial t}(\rho u) + \nabla(\rho u V) = -\frac{\partial p}{\partial x} + F_{vis} + F_{body}}$$

↳ differential form

→ Corollary 1: If the flow is steady and no body forces are acting the reduced form of above equation becomes,

$$\boxed{\oiint_S (\rho\vec{V}d\vec{s})\vec{V} = -\oiint P\cdot d\vec{s} + F_{viscous}}$$

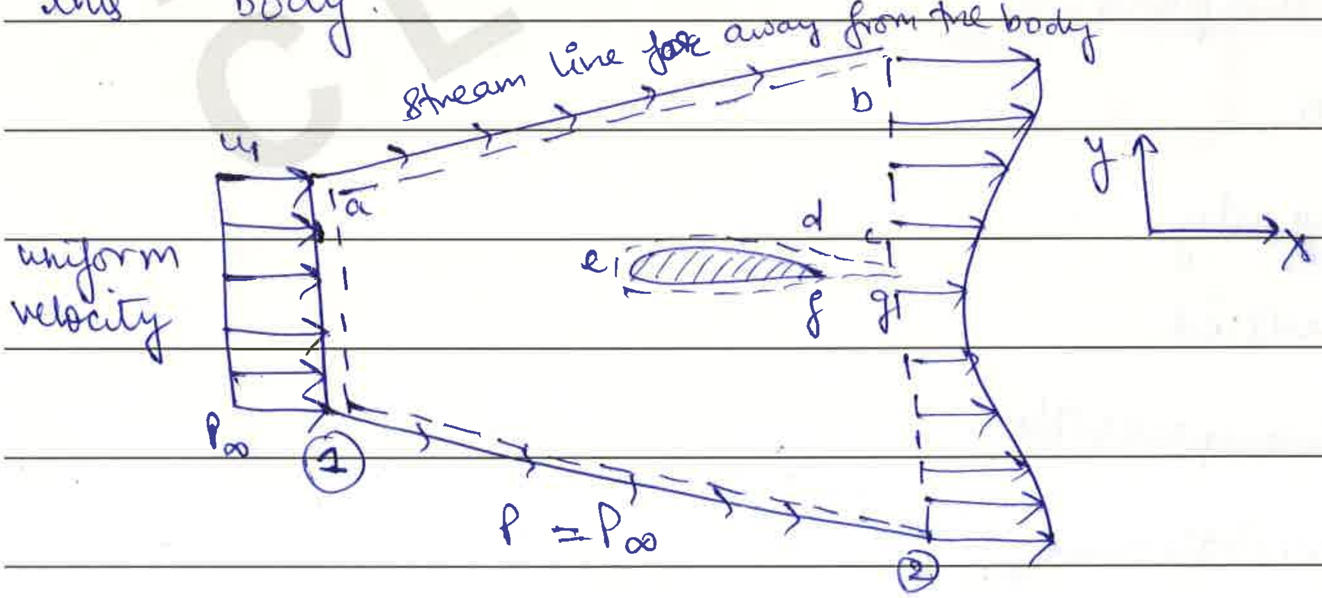
→ Corollary 2: If the flow is steady, inviscid and no body forces are acting the reduced form of Navier Stokes,

$$\boxed{\oint\!\!\oint_S (\rho \vec{V}\cdot d\vec{s})\vec{V} = -\oint\!\!\oint_S P\,d\vec{s}}$$ → Integral form

$$\left.\begin{aligned} \nabla(\rho u V) &= -\partial P/\partial x \\ \nabla(\rho v V) &= -\partial P/\partial y \\ \nabla(\rho w V) &= -\partial P/\partial z \end{aligned}\right\}$$ – differential form

→ Applications of momentum equation (Drag of 2D body)

→ Consider a 2D body in a flow as sketched in figure a control volume is drawn around this body.



$$\boxed{D = \int_h^b \rho_2 u_2 (u_1 - u_2)\, dy}$$ steady, no body force, 2D

→ For incompressible flow 'ρ' is constant and is known,

$$\boxed{D = \rho \int_h^b u_2 (u_1 - u_2)\, dy}.$$

which is applicable for 2D steady incompressible flow with no body force.

→ Ideal potential flow (2D) :

→ Assumptions:

① 2D

② Steady

③ inviscid

④ Incompressible

⑤ Irrotational

→ By continuity equation,

$$\cancel{\frac{\partial}{\partial t}\iiint_{\forall} \rho \, d\forall}^{\,0} + \oiint \rho \vec{V} \, d\vec{S} = 0$$

$$\therefore \boxed{\oiint_S \rho \vec{V} \cdot d\vec{S} = 0}$$

→ For incompressible flow, $\rho = c$.

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0}$$ → continuity equation for steady, incompressible flow

$$\boxed{\nabla \cdot \vec{V} = 0}$$

→ NOTE:

Particles of the flow must rotates about its own mass centre to be called as rotational flow.

→ Rotation:

It is the angle of velocity of particle about its centre of mass.

Rotation vector $\vec{\omega} = \omega_x\hat{i} + \omega_y\hat{j} + \omega_z\hat{k}$

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$\omega_x = \frac{1}{2}\left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right]$$

$$\omega_y = \frac{1}{2}\left[\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right]$$

→ Vorticity $= \vec{\zeta} = \nabla \times \vec{V} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$

$$\vec{\zeta} = \hat{i}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) - \hat{j}\left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right) + \hat{k}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$= \hat{i}\,2\omega_x - \hat{j}(-2\omega_y) + \hat{k}(2\omega_z)$$

$$= 2(\omega_x\hat{i} + \omega_y\hat{j} + \omega_z\hat{k})$$

$$\boxed{\nabla \times \vec{V} = \vec{\zeta} = 2\vec{\omega}}$$

vorticity = 2 × rotation

→ <u>Note:</u> If curl of velocity vector $\vec{\zeta} = 0$, the flow is said to be irrotational.