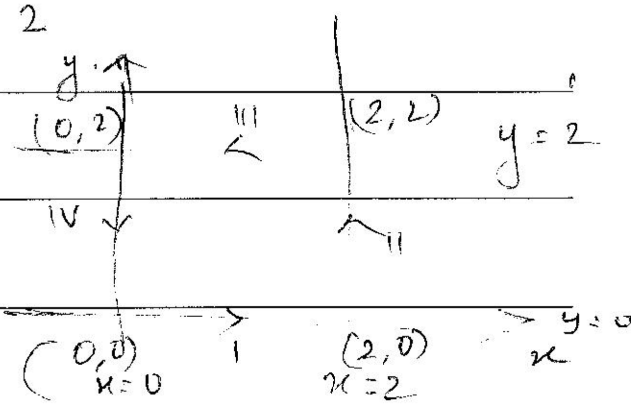
Math Solutions to GATE 2017 AE

(Q1) The vector $\perp$r to 2 vectors is given by their cross product. Option (D) $\dfrac{\vec{v}_1 \times \vec{v}_2}{|\vec{v}_1 \times \vec{v}_2|}$ is the unit vector for $\vec{v}_1 \times \vec{v}_2$. Thus Option (D).

(Q2) Given $I = \int_c (x-y)\,dx + x^2\,dy$, where c is boundary given by ~~ccu~~ $0 \le x \le 2$; $0 \le y \le 2$



Path I: $y = 0$, $dy = 0$

Thus $\therefore I = I_1 = \int_0^2 x\,dx = 2$

Path II: $x = 2$, $dx = 0$

$\therefore I = I_2 = \int_0^2 4\,dy = 8$

Path III: $y = 2 \Rightarrow dy = 0$

$\therefore I = I_3 = \int_0^2 (x-2)\,dx = 2$

Path IV: $x = 0 \Rightarrow dx = 0$

$\therefore I = I_4 = \int_0^2 0\,dy = 0$

$\therefore I = \sum I_i = 12$

(Q3.) $\vec{v}(t)$ is a unit vector and represented as parametric eqn of t. Hence derivation of it w.r.t 't' represents gradient which is always ⊥r to original vector. Thus

$$\vec{v} \cdot \frac{d\vec{v}}{dt} = 0$$

(Q4.) $y^2 + K^2\lambda y = 0 \Rightarrow \left[\frac{\partial^2}{\partial x^2} + K^2\lambda\right]y = 0$

$\Rightarrow r^2 + K^2\lambda = 0$

$\Rightarrow r = \pm K\sqrt{\lambda} \Rightarrow y = A\cos(K\sqrt{\lambda}x) + B\sin(K\sqrt{\lambda}x)$

~~and $y' = -A\sqrt{\lambda}\sin(K\sqrt{\lambda}x) + B\sqrt{\lambda}\cos(K\sqrt{\lambda}x)$~~

Applying IVs:

(i) $y(0) = 0$

$\Rightarrow A\cos(K\sqrt{\lambda}x) + B\sin(K\sqrt{\lambda}x) = 0$ when $x = 0$

$\Rightarrow \boxed{A = 0}$

Also, $y(1) = 0$

$\Rightarrow B\sin(K\sqrt{\lambda}x) = 0$ when $x = 1$

$\Rightarrow K\sqrt{\lambda} = n\pi$

$\Rightarrow \lambda = \left(\frac{n\pi}{K}\right)^2$

$\therefore y_n = \sin(n\pi x)$ Option (C).

(Q5.) $\int_{-1}^{1} f(x)\,dx = \sum_{j=1}^{3} A_j f(x_j)$

$I = A_1 f(x_1) + A_2 f(x_2) + A_3 f(x_3)$

$= \frac{8}{9} f(0) + \frac{5}{9} f\left(-\sqrt{3/5}\right) + \frac{5}{9} f\left(\sqrt{3/5}\right)$

This formula is valid for $x^0, x^1, x^2, x^3, x^4, x^5$ which is Option (B).

(Q26.) $A = \begin{bmatrix} 2 & 0 & 2 \\ 3 & 2 & 7 \\ 3 & 1 & 5 \end{bmatrix}$; $B = \begin{bmatrix} 4 \\ 4 \\ 5 \end{bmatrix}$

det $A = 0$

$\Rightarrow$ $Ax = B$ has infinite Solutions or No solution

To check we calculate the rank of the Aug. matrix

if rank Aug matrix $>$ rank of co-eff matrix, no sol^n exists.

if rank of Aug. matrix $==$ rank co-eff matrix, Unique sol^n else infinite sol^{ns} exist.

Thus, $[A:B] = \left[\begin{array}{ccc|c} 2 & 0 & 2 & 4 \\ 3 & 2 & 7 & 4 \\ 3 & 1 & 5 & 5 \end{array}\right] \xrightarrow{\text{Row Echelon}} \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\because$ Rank of $[A:B]$ is less than equal to rank A,

& det A = 0 $\Rightarrow$ infinite sol^{ns} exist.

Note:

rank $[A:B]$ = rank $[A] = 2$ and it is less than no. of unknowns i.e 3. Thus infinite sol^{ns}. $\boxed{\text{sol}^n \ (B)}$

(Q 27) $A = \begin{bmatrix} 2 & -6 \\ 0 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$K = x^T A x = [x_1 \ \ x_2] \begin{bmatrix} 2 & -6 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 2x_1^2 - 6x_1x_2 + 2x_2^2$

$= 2(x_1^2 + x_2^2 - 3x_1x_2) = 2\left[\underbrace{(x_1 - x_2)^2}_{+ve} - \underbrace{x_1x_2}_{\text{can be anything}}\right]$

Thus $x^T A x$ can be anything.

(Q 28) $y'' + 4y' + 6y = f(t)$; $y(0) = 2$, $y'(0) = 1$

Taking Laplace Transforms on both sides

$\left[s^2 Y(s) - s\,y(0) - y'(0)\right] + 4\left[sY(s) - y(0)\right] + 6\,Y(s) = F(s)$

On solving, ~~Simplifying~~: $Y(s) = \dfrac{F(s) + 2s + 9}{s^2 + 4s + 6}$

(Q29.) $\frac{\partial^2 u}{\partial t^2} - 25\frac{\partial^2 u}{\partial x^2} = 0, \quad 0 < x < 1$

$u(x,0) = 0.01 \sin(10\pi x)$

$\frac{\partial u}{\partial t}(x,0) = 0; \quad u(0,t) = 0, \; u(1,t) = 0$

Above is the wave eqn where $l = 1$,

$$u(x,t) = \sum_{k=1}^{\infty} \sin(k\pi x)\left[\alpha_k \cos(ck\pi t) + \beta_k \sin(ck\pi t)\right]$$

$$u(x,0) = \sum_{k=1}^{\infty} \alpha_k \sin(k\pi x), \quad \frac{\partial u}{\partial t}(x,0) = \sum_{k=1}^{\infty} \beta_k ck\pi \sin(k\pi x)$$

$\therefore u(x,0) = 0.01 \sin(10\pi x)$

$$\Rightarrow \alpha_k = \begin{cases} 0.01; & k = 10 \\ 0; & k \neq 10 \end{cases}$$

$\frac{\partial u}{\partial t}(x,0) = 0 \Rightarrow \beta_k = 0$

$$\therefore u(x,t) = \sum_{k=1}^{\infty} \sin(k\pi x)\, \alpha_k \cos(ck\pi t)$$

$$= 0.01 \sin 10\pi x \cos 50\pi t \qquad \left[\because c^2 = 25 \Rightarrow c = 5\right]$$

$$u(0.25, 1) = 0.01 \sin \frac{10\pi}{4} \cos 50\pi$$

$$= 0.01 \sin \frac{\pi}{2} \cos 50\pi$$

$$= 0.01 \times 1 \times (-1)^{50} = 0.01$$

(Q30.) $x^2 \frac{d^2y}{dx^2} + 5x \frac{dy}{dx} + 4y = 0$

Cauchy Euler's Eqn. when we put $y = x^\alpha$ in abv eqn.

So,

$$\alpha(\alpha-1) + 5\alpha + 4 = 0$$

$$\alpha^2 - \alpha + 5\alpha + 4 = 0$$

$$\alpha^2 + 4\alpha + 4 = 0$$

$$(\alpha+2)^2 = 0 \Rightarrow \text{roots are real repeated}$$

$$\alpha_1 = \alpha_2 = -2$$

Thus y would be of the form:

$$y = C_1 x^{-2} + C_2 \ln(x) x^{-2}$$

which contains -ve Int. in x and logarithmic func of x. Hence option (C).